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Power Laws

Critical Equation #5 for Business Leaders

$Y = AX^b$

Overview

Every business leader is familiar with the Pareto Principle, aka the 80/20 rule or Juran's role of the vital few. Ziph's law, related to distribution or frequency of words in any language, and even Benford's law of 1s, widely used in fraud detection, are related to Pareto's logic. All are functions within the statistical family of power laws, the topic of our Critical Equation #5 for Business Leaders. Power laws state that large occurrences are rare and small occurrences are common. The degree of rarity vs. commonness is crucial to our discussion.

The classic and oft-referenced example of the 80/20 rule is when 80% of profits come from 20% of products. The importance is not so much in 80/20, but rather that we rarely observe 50/50. Historically, managers who understand this apparent imbalance (often called the Gini index) are able to focus on what really matters and hence better allocate scarce resources. In the online world, early movers have successfully created differentiation in the long tail (very low probability event with significant consequence in a risk framework). For example, online book sellers have been successful selling products rarely found in retail stores due to lower carrying costs of storage and distribution.

We also know that these principles apply not only to the business world but also to the physical world, pointing to the universality of power laws. Another long-tail area that has recently evolved because of technology is "Culturomics," which focuses on the linguistics and cultural and societal change (see Google Trends @ Google Labs).

Without an understanding and application of power laws, meeting commitments, which has been at the heart of our other Critical Equations for Business Leaders, can be problematic. How many times have you missed your financial commitments (sales, operating margin, net income, cash flow forecasts) because you did not consider a wide range of alternatives in the planning and budgeting process? Odds are, your forecasting tools did not allow for the outliers or long tails inherent in power laws.

Over the past few years, we have seen an unprecedented increase in debate and discussion in board rooms and halls of academia on the relevance of Black Swans (positive and negative), outliers, data mining, variance in variance (i.e., volatility in the risk itself), and resurgence in scenario planning from the 1970s. Many companies have adopted Enterprise Risk Management (ERM) at an increasing rate, and Chief Risk Officers are coming into vogue. Ambiguous environments of the past decade have, in many instances, made assumptions such as normality in distributions close to useless in forecasting.

Business leaders have long understood the importance of simplicity. What may not have been obvious is that, within reason, simplification (i.e., reducing the number of customers, suppliers, products, etc.) is a primary implication of power laws. Of course, this should not be taken literally to mean dropping 80% of your customers in an 80/20 relationship. Understanding variable and fixed costs is essential before any reduction.

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Recent advances in Enterprise Performance Management (EPM), as well as real-time data, may help make proper cost breakdowns. Power laws can, however, tell us something about the importance of simplification. At the very least, simplicity will lead to faster decision making, reduced complexity, and self-confidence from having more under your control.

In our Critical Equation #5 for Business Leaders, we will introduce and review:

- the basic mathematics of power laws
- common applications in business as well as nature, science and society
- a comparison of power laws to normal distributions
- power laws in communications
- experience curves and power laws
- GRPI and power laws, and
- implications for managers.

The power of understanding power laws is the potential for enhanced profitable growth.

TRI's Critical Equation # 5 – Power Laws

Our equation # 5, the power law function, is represented by

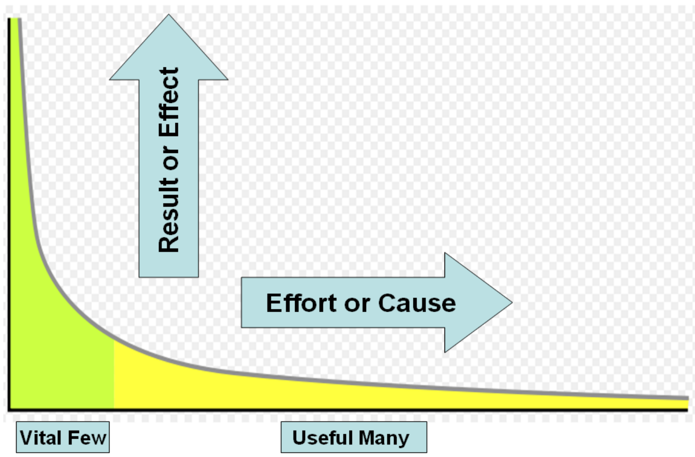
$$Y = AX^b$$

where, in a typical business application, Y is the result or effect (the solution to the computation on the Y axis), A is a constant that defines the size, scale, or magnitude of a the process, X is the effort or cause (on the X axis) and b is the constant characteristic number that defines the relationship between Y and X.

Exhibit 1 is a typical graphical portrayal of a power law. Note the area identified as the "vital few" and "useful many." While not a necessity in power law mathematics, in most business applications b is a negative number, shown by the negative slope in Exhibit 1. You also can see the non-linearity that many authors identify with numerous processes.

Exhibit 1

Power Law Graph



The primary implications of Exhibit 1 are:

A small percentage of efforts provide the majority of results.
A large percentage of efforts provide a small amount of results.

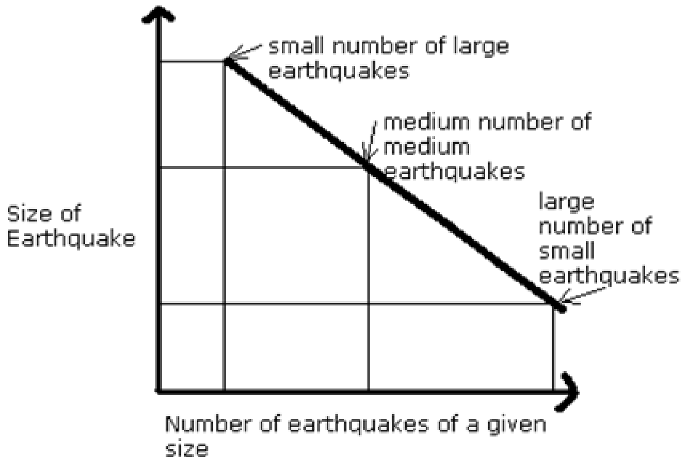
In statistical applications, a common practice to determine if a power law is appropriate to describe a process is to use the properties of logarithms to convert an equation to a straight line where standard linear regression can be applied and the coefficient of determination can be used to assess the appropriateness of a power law relationship. This log-log specification for our equation #5 is given by

$$\log(Y) = \log(A) + b \log(X)$$

In Exhibit 2, a linear example of power laws is applied to earthquakes, one of the more common and discussed applications of power laws to natural disasters.

Exhibit 2

Linear Example of Power Law



Representative Applications of Power Laws

Exhibit 3 is a sample of common applications of power laws in nature, science and society as well as business. These lists are not exhaustive, but the universality and applicability of power laws to a wide variety of areas should be obvious.

Exhibit 3

Sample Common Applications

Nature, Science & Society

- Wealth and Population
- Distribution of Words in any Language
- Earthquake Intensity
- Power Blackouts
- Cities and Population
- Patients who use Health Care
- Heartbeats
- Criticality in Sand Piles
- Criminals and Crime
- Time in Traffic Jams
- Pollution
- Oil Field Size
- Forest Fire Destruction
- Meteor Devastation
- Forgetting Curve
- Number of Species in an Environment
- Etc, Etc

Business

- Volatility in Industrial Production
- Bankruptcy Filings by Size
- Customer Profitability
- Sales Distribution by Product/Service
- Customer complaints
- Marketing Efforts
- Total Quality Control and Six Sigma
- ABC Analysis (Inventory Impact)
- Value of a Meeting's Duration
- Software Fixes and Error Reduction
- Aspects of Insurance and Actuarial Science Severity
- Session times on a Website
- Learning Curve
- Success of Early Online Retailers (Long Tail in Digital Economics)
- Movie and Pharma Blockbusters
- Etc, Etc

To Identify Only A Few

Both sides of Exhibit 3 are relevant to any business leader. Think about the importance of understanding probability and severity in the world of insurance. Our global capital markets respond to both. The power law impact of societal change defines many aspects of long-range strategic planning and future business opportunities and is fundamental to the application of Political/Economic/Social/Technological analysis or PEST. PEST analysis combined with Strength/Weakness/Opportunity/Threat or SWOT analysis is central to forecasting sales in a volatile and uncertain world. Both PEST and SWOT can be linked to power law dynamics.

Normal Distributions Versus Power Laws

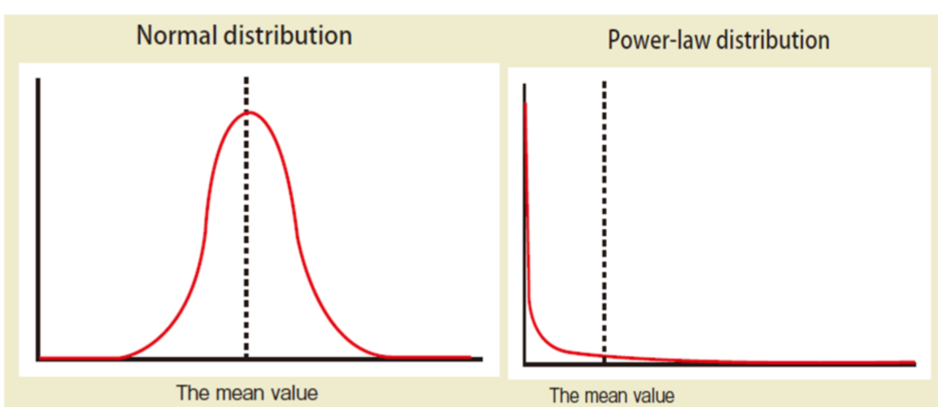
Many applications of normal, bell-type curve distributions, which we all grew up with and still are the basis of our academic management development programs, simply do not apply. Since the economic turmoil of 2007-2010, normal distributions are out and outliers in. For decades, a few authors attacked normal distributions as applicable to understanding returns on equities in our capital markets. In fact, many have said, “The downfall of the fat cats is the fat tails.”

Normality assumptions in the financial service world came under attack with the downfall of Long-Term Capital Management in the late 1990s. Even before this, Black Monday on October 19, 1987, should have opened the eyes of the investment community as equity markets globally were devastated (e.g., the Dow 30 dropped 23% in one day). Normality simply would not have recognized this scenario. Even Value at Risk (VaR), widely used in financial services as a risk measure, may have given false security and misleading signals to many traders and portfolio managers in the recent economic downturn. Many authors would lead you to believe that, in its basics, VaR may be easy to understand, especially in the graphical portrayals we often see. VaR, however, is very dangerous when misunderstood, and economic reality does not fit those nice bell curves.

Exhibit 4 shows a normal distribution (left) and a power law (right). In the world of power laws, the mean value (expected value) may not exist, and variances may be infinite, thus negating all of the underlying assumptions we use in normality. Infinite variance, were the variance cannot be defined, can play havoc with forecasting.

Exhibit 4

Normal Distribution Compared to Power Laws



Elements of basic logic dictate that certain items will follow normal distributions. Normal distributions typically apply within processes that are large in number and independent. For example, modeling the risk in rolling a die or flipping coins will follow a normal distribution. The issue of independence is the Achilles' heel of normal distributions in a complex and inter-related world. If people's heights followed a power law, we would have a few people 1000 feet tall with the vast majority of the world's population at 1 foot. On the other hand, the aforementioned word frequencies cannot possibly follow a normal distribution. If they did, most words would have fairly equal frequencies, and outliers would be extreme. For example, the word ***"Gaussian,"*** an expression for a normal distribution, could be in every other sentence.

Normal distributions however are not appropriate when modeling complex systems where the failure of one component can cause others to fail and start a cascade-type propagation. Skewness in costs and/or in capacity (righ- skewed in costs and left-skewed in capacity), non-negative numbers, such as the distribution of price, typically would not fit normal distributions. Cost overruns and project delays can be related to the non-linearity of a power law distribution. Interestingly, in our experience, many Monte Carlo simulations such as Crystal Ball and/or @ Risk do not have power law distributions as modeling alternatives. When forecasting ranges of outcomes, this can lead to erroneous estimates and increase the probability of missing commitments. Power laws have application whenever the potential cascade risk in complex systems is likely.

To fairly model a cascade effect in a Monte Carlo simulation, we must assume a correlation structure (i.e. the degree of a relationship betweens two variables). The correlation structure itself may not be stable. It is the dependence structure that causes the cascade. Even in situations where users recognize non-normality of the distribution process, we rarely see proper use of a correlation structure. The end result is that the variance of the potential range of outcomes is minimized relative to reality. The primary reason we see this result, particularly in budgeting, is that cross-functional teams are not involved in the process. Too much of the budget is left to finance. The overall systemic risk may be ignored, and overall ranges of potential outcomes are reduced. Everyone during the subsequent business review is left wondering why we had such a large variance to plan. Bottom line, not considering outliers in long tails can lead to poor risk management as well as inferior forecasting.

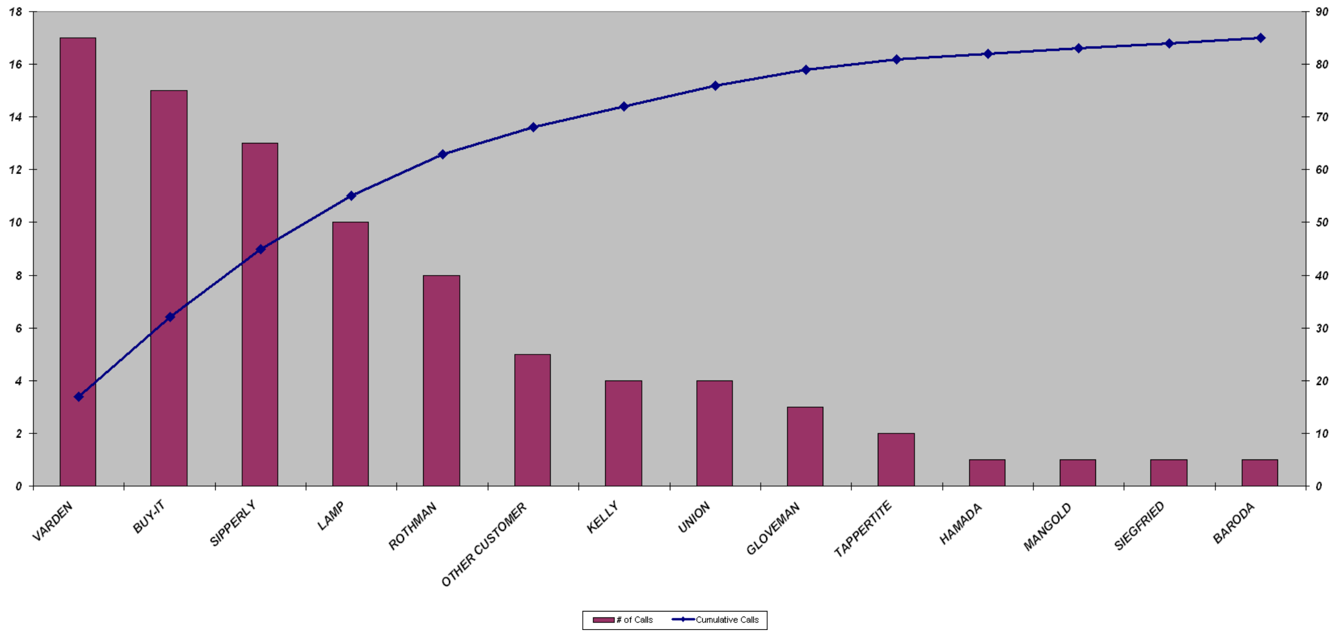
Remember, outliers can be good and bad. Acknowledging power laws and outliers will not necessarily guarantee you will predict the next so-called Black Swan event because they are inherently unpredictable (ex-post, of course, it is easy to think we have an explanation when in fact none existed), but you will be better prepared to deal with and/or mitigate the impact of these events on your business plus possibly give better guidance to your Board and or investors.

Communication and Power Laws

TRI uses role play in its business simulations to build negotiation and communication skills in leaders. Participants learn how to create value from communication. Exhibit 5 is the typical pattern we observe over thousands of participants in terms of the role play characters they choose to call. A classic Pareto. Like reality, most value comes from information gleaned from a few select characters in the simulation. This, however, should not imply that speaking to the outliers has no value. Our clients often are surprised by the Pareto effect in the role play but quickly recognize that, if the calls are limited, a call strategy is necessary because communication becomes a scarce resource.

Exhibit 5

Role Play Distribution



In reality, none of us have the time to communicate with everybody we might like to and/or get all of the information we need prior to making a decision.

Experience Curves and Power Laws

In businesses with large volumes and complicated product/service portfolios, power laws have application in the form of experience curves. These applications are critical to successful cost forecasting and pricing on large-volume opportunities. Exhibit 6 shows standard experience curves in non-linear and linear space. An 80% learning curve means that a 20% cost reduction should occur for every doubling of output. Typical reasons for learning curve effects are: labor productivity gains, manufacturing process efficiencies, improved design of the product, and reduced selling or distribution costs.

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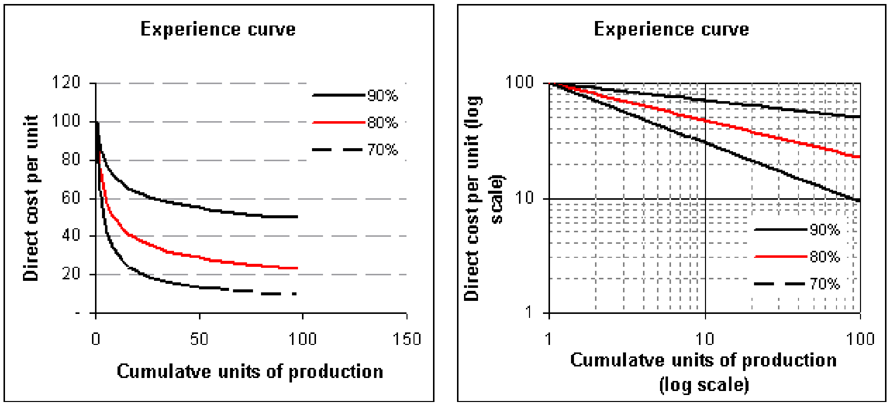
Exhibit 6

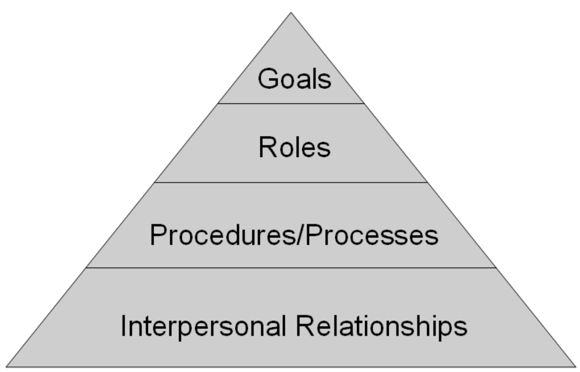


GRPI and Power Laws

The GRPI model is widely used in executive education as a paradigm for effective team development and for understanding the drivers and processes critical to making decisions and meeting commitments. It starts with setting proper goals, establishing clear roles, determining processes for operational excellence, and, finally, understanding how people work together. The GRPI model often is in the form found in Exhibit 7.

Exhibit 7

GRPI



What often is not discussed in the applications of GRPI is the fact that a power law effect is at work. This has direct implications for a business leader's ability to deliver on a commitment. As illustrated in Exhibit 8, numerous studies have shown that an approximate 80/20 Pareto effect is at work. The bottom line is that 80% of missed commitments can be attributed to misspecification of goals; 80% of the remaining 20% (or 16%) to improper roles, etc.

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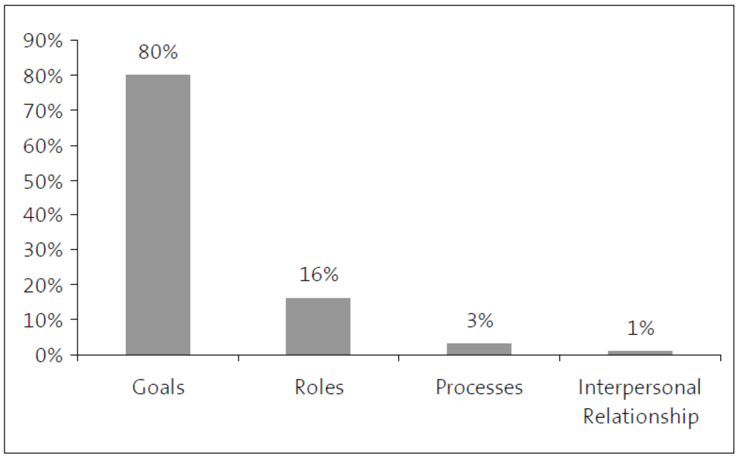
Exhibit 8

GRPI and Pareto



The crucial learning here is that uncertainty at one level gets transmitted down. For example, misspecification of goals coupled with unclear roles is a disaster in the making. The best processes and interpersonal relationships rarely will save the day. The power law effect is the cascade that can occur without clarity at each level. This is the analogy to the well-known collapse of a pile of sand and can help explain rapid corporate deterioration as well as bankruptcy. It is the slippery slope that is very hard to escape.

Summary

Power laws state that large occurrences are rare and small occurrences are common. They are ubiquitous not only in business but also in numerous aspects of nature, society, and science. Understanding power laws allows managers to focus on what really matters and how to better allocate scarce resources. With the glut of information we face and an environment where cause and effect is harder than ever to decipher, simplicity is crucial to all business models. Simplicity can come from an understanding of power laws. This understanding also may enable us to better deal with the inevitable outliers.

Your ability to meet commitments (on operating margin, net income, cash flow and return on investment as well as non-financial measures) in a volatile and ambiguous world may be improved by applying power laws to your forecasting and making them part of your every thought. There is power in understanding power laws. What are the long tails in your business?

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