TRICorporation

Experiential Leadership and Simulation Programs

Improved decision-making. Enhanced performance. Exceptional results.

Bayesian Probability

Critical Equation #7 for Business Leaders

$$P(A|B) = [P(B|A)*P(A)]/P(B)$$

Introduction

All business leaders face risk and uncertainty when making decisions and investments. Whether explicit or implicit, all decision makers encounter scenarios that require an assessment of the probability of success. The probability of winning a bid is one of the most common of these scenarios. Proper information is critical to success in making decisions. We all are aware that information comes from varied sources. Rarely do we have perfect and complete information. Typically, the higher the quality of information, assuming correct interpretation of course, the better decision making under uncertainty. Statistics and probability, when properly applied, can be excellent sources of information that can yield more competitive decisions. The purpose of our Critical Equation #7 is not to make you an applied statistician, but to help you gain insight into the drivers of improving the estimation of probability, hence improving your decision-making and critical-thinking skills. Thinking in terms of probability can be very hard.

A key to competitive advantage may be to be ***"Less Wrong."*** Our goal is to provide you a tool to be ***"Less Wrong."*** Consider observing a game of dice. Most of us will know that, given a fair die, the number four will come up one out of six times in the long run. One out of six is approximately 17% and probably would be an intelligent estimate of the probability. If, however, over the span of a few days of rolling the die, the number four was occurring at a much higher or lower rate, it would not be unusual to adjust expectations due to the arrival of new information. The simple explanation, of course, is that the die may not be fair. True believers in frequency theory, however, would not adjust their expectations and base their decision on the fact that long runs of any number are not unusual given enough time in play. Non-believers will use the arrival of information to adjust their probability estimates going forward. (and naturally be continually aware that new information may force future readjustment). The decision-maker who incorporates the new information often is called a ***"Bayesian."***

Poker is a classic example of applied probability. The Bayesian would attempt to understand not only the rational quantitative dimensions of the game but also how the probabilities are altered as a result of the emotional reactions of competitors. An HR business example would be the manager who has interviewed a candidate for a sales job and has made an assessment of a low probability of success. However, when the candidate's high scores on the company's sales assessment tool are revealed to the HR manager, the prior low probability of success might be revised upwards. The arrival of new information can influence all of us to varying degrees, of course, because of personal bias. The HR manager may be a ***"Bayesian"*** and not even know it.

Bayesian probability is the essence of our Critical Equation #7 for Business Leaders. Bayesian probability is credited to the Reverend Thomas Bayes in the 1700s and, naturally, has been extended by many others over the ensuing centuries. Bayesian probability typically is expressed as a percentage and is an estimate of the likelihood of the occurrence of an event. In Bayesian probability, an R&D scientist has an initial set of beliefs or expectations that typically will change as new information is derived from the research process. In this paper, rather than focus on probability theory, we will try to bring common sense and application to what many view as abstract and prefer to ignore.

Our equation #7 has applications in many aspects of business and society in general. One important area is in calculating the probability that a positive test is in fact a false positive. The success of drug research constantly can be reassessed with Bayesian probability that can reduce both the cost and time to develop a new drug. Extractive industries interested in the probability of a “hit” can find significant applications for our equation #7. And, chances are, the spam filters on your e-email make use of Bayesian probability. Otherwise your inbox would be cluttered with even more junk. Many authors have applied Bayesian probability to court cases (think of the tradeoff between false positive and false negatives and the concept of beyond a reasonable doubt).

Bayesian probability, like many relationships, has its pros and cons, its advocates and detractors. The key in any application is to understand the limitations and when the tool can be a useful guide to improved decision making. The next section introduces the mathematical expression or equation for Bayesian probability. We then provide some practical applications, discuss the importance of creating a Bayesian culture (similar to our Options culture of Critical Equation #6), and re-examine our Monty Hall problem from equation #6 on options in the framework of Bayesian probability.

Critical Equation # 7 – Bayesian Probability

Our equation # 7, Bayes’ Theorem, is represented by

$$P(A|B) = [P(B|A)*P(A)]/P(B)$$

Where P is probability, A and B are two events, and **“|”** means given. $P(A)$ is the probability of event A occurring and is often referred to as the prior probability. $P(B)$ is the probability of event B occurring. $P(A|B)$ is the probability of A given that B has occurred and is often referred to as the posterior probability. This is not necessarily the same as $P(B|A)$. $P(A|B)$ is also commonly read as the conditional probability of A given that B has occurred.

The computational form of equation # 7 is given by:

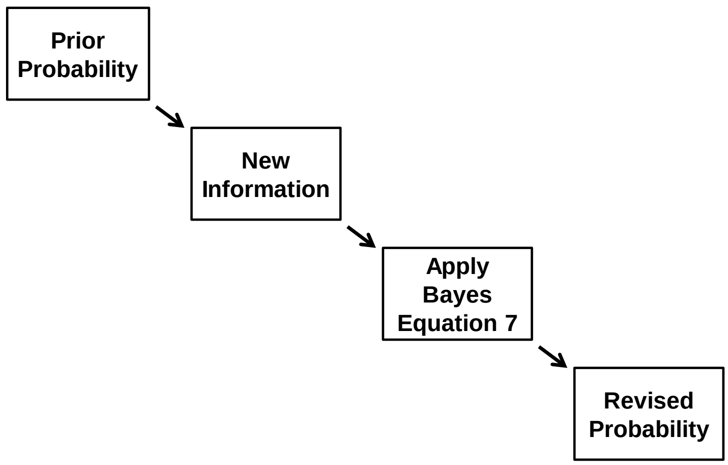
$$P(A|B) = \frac{P(B|A)*P(A)}{P(B|A)*P(A) + P(B|\text{Not } A)*P(\text{Not } A)}$$

where $P(\text{Not } A)$ is the probability of A not occurring or $1 – P(A)$. A careful examination of the denominator of equation #7 is that it reflects all outcomes of event B given that event A does and does not occur.

The logic of equation #7 is seen in Exhibit 1.

Exhibit 1

Flow of Bayes' Probability



Practical Applications

In this section we present three applications of equation #7 as well as a practice example from the financial services sector.

- Application 1 – Determining probability of defects from suppliers
- Application 2 – Estimating false positives and negatives in medical testing
- Application 3 – Forecasting the probability of a win in a down select

Our first application is found in Exhibit 2. The initial data shows that we have two suppliers of the same raw material with the quantities of both given. Two suppliers could exist as a backup strategy or to manage capacity issues. The data is presented as frequencies in the left table and converted into percentages in the table on the right. It would appear that if we sampled a piece of raw material the probability of its coming from supplier A is 20%. This is a correct assumption based on the given frequencies. If, however, we sampled one piece of raw material and it was defective, what is the probability it is from supplier A? In Exhibit 2 you will note the decision tree with the possible outcomes.

Exhibit 2

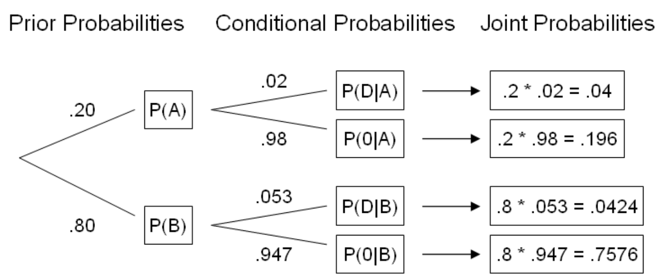
Application of Bayesian Probability 1

Frequency

	Supplier A	Supplier B	Total
OK	98	379	477
Defective	2	21	23
Total	100	400	500

Percentages

	Supplier A	Supplier B	Total
OK	19.6%	75.8%	95.4%
Defective	.4%	4.2%	4.6%
Total	20.0%	80.0%	100.0%



In Exhibit 3 we substitute the values from Exhibit 2 into equation #7. Note the probability of the defective material's being from supplier A is 8.7%, very different from the 20% probability of its being from supplier A. If we were trying to allocate the cost of defectives, we might simply assign 20% to supplier A, when in fact, the true value would approximate the 8.7%. The additional information that it is defective from the sample provides a significant shift in the true probability.

Exhibit 3

Application of Equation 7

$$P(A|D) = \frac{P(D|A)*P(A)}{P(D|A)*P(A) + P(D|B)*P(B)}$$

$$P(A|D) = \frac{.02*.20}{.02*.20 + .053*.8} = .087 \text{ or } 8.70\%$$

8.70% vs. 20.0% with New Information

Our second application, often seen in medical testing, is in Exhibit 4. The initial data shows that 1% in a sample patient population of 100,000 has a particular disease. There is a test procedure to determine if a patient has the disease. The test, of course, while very good is not perfect. As seen in our first application, the data is converted into percentages in the table on the right. It would appear that that the probability of a patient's having the disease is 1%. This is a correct assumption based upon the given percentages.

The tricky part for consideration is that 3% of patients without the disease will actually test positive (here T+). Because 99% of patients do not have the disease, the probability is 3% divided by 99% or 3.0303%, what is classically known as a false positive. The false negative, another classic statistical type error, is when the patient tests negative (that is, does not have the disease) when in fact they do. False positives can create significant short-term stress, while false negatives could be life threatening in this example. There is a tradeoff between the two. If we sampled one patient and the test indicated a positive, what is the probability the disease is truly present?

Exhibit 4

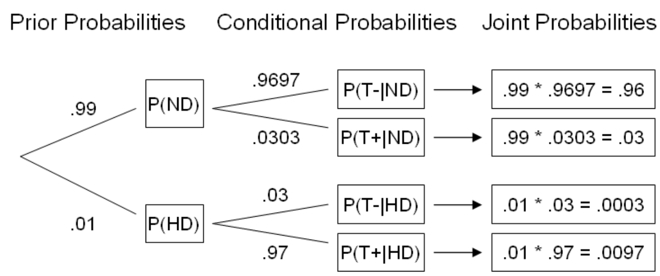
Application of Bayesian Probability - 2

Frequency

	No Disease ND	Has Disease HD	Total
Test Neg T-	96000	30	96030
Test + T+	3000	970	3970
Total	99000	1000	100000

Percentages

	No Disease ND	Has Disease HD	Total
Test Neg T-	96.0%	.03%	96.03%
Test Pos T+	3.0%	.97%	3.97%
Total	99.0%	1.0%	100.0%



In Exhibit 5, we substitute the values from Exhibit 4 into equation #7. Note the probability of the patient's truly having the disease is approximately 24%, relative to the 1% of our sample that has the disease. This could result in a serious misuse of statistics.

Exhibit 5

Application of Equation 7 - 2

$$P(HD|T+) = \frac{P(T+|HD)*P(HD)}{P(T+|HD)*P(HD) + P(T+|ND)*P(ND)}$$

$$P(HD|T+) = \frac{.97*.01}{.97*.01 + .0303*.99} = .244 \text{ or } 24.4\%$$

24.4% vs. 1.00% with New Information

Exhibit 6 shows our third application, often seen in bidding proposals. These bidding proposals in RFPs/RFQs could be for large commercial or government customers. The overall history of winning bids, given the combination of qualified competitors, is 55%. We know that winning companies have a 75% chance of being down selected and that 40% of unsuccessful companies are actually picked in the down select process. If we have just been down selected, what is the revised probability we will win? In Exhibit 6, ***W*** is win, ***NW*** is not win, and ***2'nd*** is the down select. Note the probability of winning increases dramatically when a company has been down selected.

Exhibit 6

Application of Equation 7 - 3

$$P(W|2'ND) = \frac{P(2'ND|W)*P(W)}{P(2'ND|W)*P(W) + P(2'ND|NW)*P(NW)}$$

$$P(W|2'ND) = \frac{.75*.55}{.75*.55 + .4*.45} = .696 \text{ or } 69.6\%$$

69.6% vs. 55.0% with New Information

Our final example is a practice problem from financial service applications of equation #7 for you to solve. Assume the following information: our historical data indicates that 80% of borrowers repay their loans in full and on time. Of the 80% that repay, 60% have a college degree. Our data also shows that 5% of those who default have a college degree. If we randomly select a borrower who has a college degree for an audit or to extend credit, what is the probability they will repay the loan? The answer can be found in the summary.

A Culture that Drives Better Estimates of Probability

In our opening paragraph we concluded that thinking in terms of probability is very hard. Study after study has confirmed this. Both individuals and teams tend to overestimate positive outcomes and underestimate negatives. In addition, once a probability estimate is formed, individuals and teams often are very slow to revise their estimates as new information arrives that is in direct contradiction to a Bayesian approach.

Social psychology has a field day with its interpretation of all the well-known biases that support poor probability estimates (conservatism, overconfidence, use of heuristics, cognitive, anchoring, representativeness and control to name a few). Post 9/11 and during the global financial crisis of the past few years, numerous authors in the risk management area have studied the impact of biases on business decisions. In our perception, the major driver is the ability or inability to control. When the needle swings to beyond your control, probability estimation, even in the absence of all other biases, becomes very difficult. Numerous authors have presented all types of recommendations on improving forecasts and their associated probability (maintain accurate records, beware of wishful thinking, break down complex events into simple tasks such as process mapping, calibration, gamble methods, Monte Carlo, prediction markets, etc).

In our Critical Equation #6, we created a corporate culture around options. We stated "***Crucial to a successful options culture is getting appropriate and accurate information in a timely manner with time to act."*** An options culture that is open to and incorporates change and new information into decision making is very consistent with one that aptly can apply Bayesian probability to its advantage. This is the reason the classic Monty Hall problem is discussed in both Critical Equations #6 and #7.

The application of Bayesian probability should promote your people to better explain their underlying assumptions and judgments and to remove the aforementioned biases. The underlying dialogue should encourage criticism and constant revision based on the arrival of new information.

In today's world of reduced guidance, estimating accurate probabilities of meeting financial commitments is crucial to both short- and long-term decision making. In TRI Corp's business simulations for corporate clients, we employ the results of a prediction market process to estimating the probability of meeting the net income plan. Our preliminary findings indicate that the greater the standard deviation of individuals' estimates within a team, the higher the probability of meeting commitments. This may be consistent with the importance of having significant divergence of opinion in decision making.

During real-world operational (business) reviews, general managers often are queried on their assessment of the probability of being able to meet their financial commitments. In Critical Equation #2 we presented a variance analysis 3-up process that also can be looked at in terms of probability of meeting commitments.

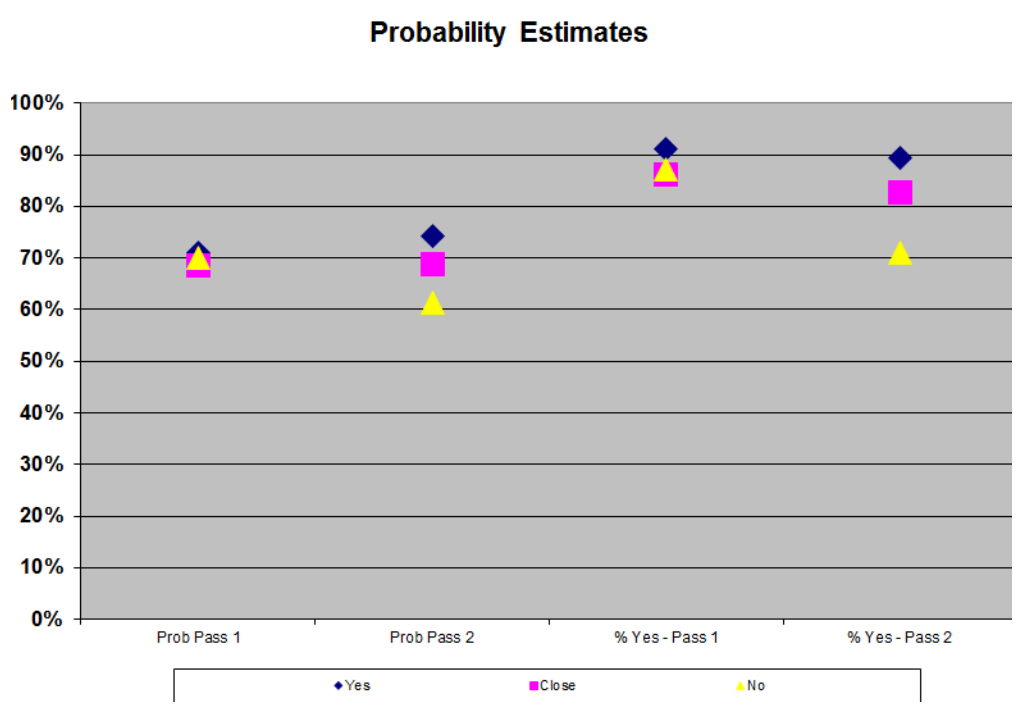
In our standard business simulations we assess the degree of confidence in financial forecasts in the second year of play. Subsequent to finalizing their budgets, each participant is asked to respond anonymously to the following questions:

1) **Will we meet and/or exceed our net income commitment for the upcoming year?**
   Yes or No

2) **What is the probability (%) we will meet and/or exceed our net income commitment for the upcoming year? __ %**

This process is repeated after the mid-year business review. Thus we have two data points per individual. Team size varies from five to seven. The average results can be found in Exhibit 7. The data set includes 96 teams for a total of 546 participants from our client companies worldwide in nearly every conceivable industry. Note: the data is segmented over three classifications. Blue represents teams that meet and/or exceed their net income commitments (23 teams), pink represents teams that are considered close, and yellow indicates teams that fall below the average (which itself is less than the minimum target).

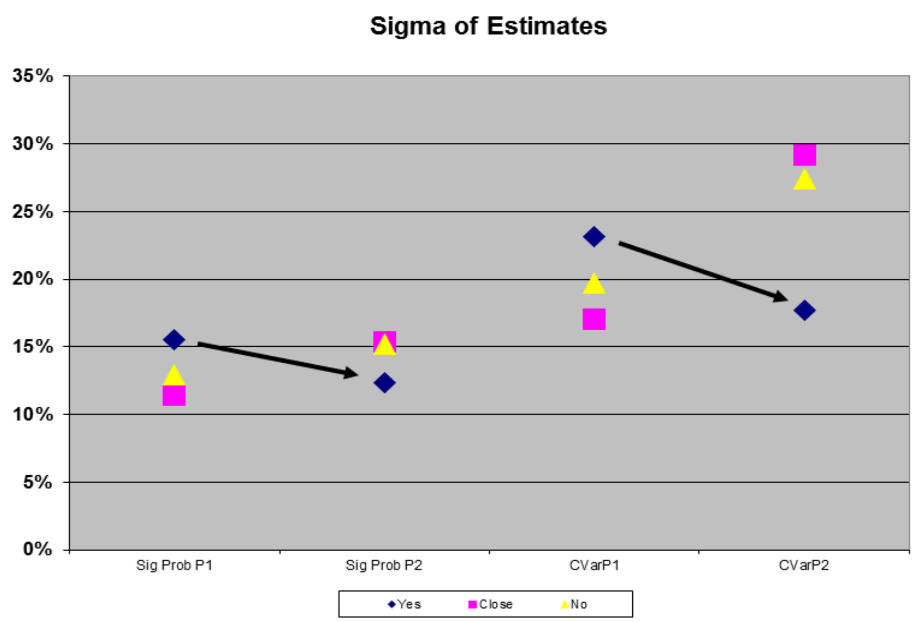
Exhibit 7



We have significant data from our simulations that shows that the prior probability of meeting and/or exceeding plan is only 25%. This information is only revealed to the program participants at the end of the program during debrief. Teams have approximately 60% to 65% complete information at a given point in time. Teams are asked to do a risk assessment of what could put their plan at risk for year 2, and this assessment is presented to the senior management reviewers. The data suggests that overestimation is common and that on average, even as mid-year results become known, reality is not incorporated. Very simply, decision making is never easy under conditions of uncertainty when one has to deal with time pressure, limited resources, limited information and divergent opinions. Limited information has the most direct correlation with the inability to estimate probability. Even the best applications of critical thinking skills can be undermined by faulty perceptions and biases. Decades ago, Benjamin Cardozo, U.S. Supreme Court Justice, said it best, ***"We may try to see things as objectively as possible. Nonetheless, we can never see them with any eyes, except our own."***

Exhibit 8 shows the standard deviation of probability estimates both at the beginning of the year and at mid-year. Interestingly, teams that ultimately meet and/or exceed plan tend to have the highest sigma that tends to decline over time. Of course, while very hard to know exactly, this does lend support to the importance of divergence of opinion in decision-making early on in a process with a smoothing effect as time goes on. While potentially anecdotal, the degree of divergence of individual opinion, again measured by sigma, does seem to increase at mid-year for teams that do not meet their commitments.

Exhibit 8



We also know from our debriefs that teams that do accurately assess their probability of meeting or exceeding plan (even in the face of a 25% prior probability) make every effort at constantly updating their position and understand what the data is telling them. While we do not strictly apply Bayes theorem as presented in our applications, we do observe in a simulated environment that teams that can make the best use of prior and new information in their decision making do on average perform very well relative to those who cannot assimilate new information as effectively.

The Value of Revealed Information and the Monty Hall Problem

In Critical Equation #6 on options, we discussed the infamous Monty Hall problem and the value of new information. Our comment was ***"For those still non-believers, please simulate it yourself but think hard about the value of new information."*** In Critical Equation #7 on Bayesian probability we are re-examining the Monty Hall problem. Exhibit 9 is the application of Bayesian probability to the Monty Hall problem. You'll recall that two goats are randomly positioned behind two doors; a car is behind a third. The assumption is the contestant wants to win the car not a goat. Thus $P(X_i)$ is 1/3. B is the event Monty opens door 3. $P(B|X1)$ = ½ because if the car is behind 1, Monty can only open 2 or 3. $P(B|X2)$ = 1 because if the car is behind 2, Monty can only open 3. $P(B|X3)$ = 0 because if the car is behind 3, Monty cannot open 3. Thus to compute $P(B)$, we weight each outcome of $P(B|Xi)$ by 1/3 probability of occurrence, which gives us the ½ in Exhibit 9.

Exhibit 9

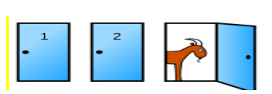
Bayes and Monty Hall

$$P(X_1) = P(X_2) = P(X_3) = 1/3$$
$$P(B) = 1/2$$
$$P(B|X_1) = 1/2,\ P(B|X_2) = 1,\ P(B|X_3) = 0$$

$$P(X_1|B) = \frac{P(B|X_1) * P(X_1)}{P(B)} = \frac{1/2 * 1/3}{1/2} = 1/3$$

$$P(X_2|B) = \frac{P(B|X_2) * P(X_2)}{P(B)} = \frac{1*1/3}{1/2} = 2/3$$

$$P(X_3|B) = \frac{P(B|X_3) * P(X_3)}{P(B)} = \frac{0*1/3}{1/2} = 0$$



Our conclusion remains the same: the switching strategy wins twice as much. There is value in incorporating new information into the decision because it impacts the probability. The corporate culture of options is very consistent with Bayesian thinking. The Monty Hall problem, as tricky as it appears, reveals the cognitive limitations all of us have in the processing of uncertainty. If you are still skeptical of the Monty Hall problem let us know.

Summary

The answer to our problem on college degrees and loan repayment is approximately 98%, rather different from the 80% before additional information. A substantial difference, and a variance that any lender needs to be cognizant of, in particular given the degrees of financial leverage found in financial services. Little losses can become catastrophes very quickly in that world.

All business leaders face risk and uncertainty when making decisions and investments. Whether explicit or implicit, all decision makers encounter scenarios that require an assessment of the probability of success. We all are aware that information comes from varied sources. Rarely will we have perfect and complete information. Typically, higher quality information, assuming correct interpretation, leads to better decision-making under uncertainty. The purpose of our Critical Equation #7 was not to make you an applied statistician but to gain insight into the drivers of improving the estimation of probability, hence improving your decision-making and critical-thinking skills. We said at the beginning, thinking in terms of probability, very simply, is very hard. A key to competitive advantage may be to be ***"Less Wrong."*** Our goal is to provide you a tool to be ***"Less Wrong."***

Bayesian probability, like many relationships, has its pros and cons, its advocates and detractors. The key in any application is to understand the limitations and when the tool can be a useful guide to improved decision making. Bayesian analysis is one of numerous ways we should be aware of to improve probability estimates.

Ask yourself, am I creating a Bayesian culture for my team? You want to promote a culture that understands social psychology biases, is continually attuned to the arrival of new information, and quick to incorporate it.

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